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Periodic Swing-By Orbits Connecting Earth and Mars

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Periodic swing-by orbits which go back and forth between Earth and Mars indefinitely are found through use of a patched conic analysis. An approach is developed which combines two round trips to Mars and two separate series of trajectories that return directly to Earth in an arrangement that is symmetric in time. The exact terminal dates for the periodic orbits are first established by computer solution in the case where the two planets are in circular, coplanar orbits; then a solution is attempted for the eccentric, inclined case. As few as four spacecraft on periodic orbits can provide fast transfers to and from Mars during every opposition period.

Introduction

THE term "periodic swing-by orbit" is taken to mean an interplanetary, free-fall (unthrusted) trajectory which visits one or more planets and revisits these same planets repeatedly for an indefinite period of time. Such orbits are the logical conclusion of multiple flyby trajectories that consist of a series of trajectory legs separated by unthrusted planetary swing-bys. Periodic orbits consist of a series of an indefinitely large number of trajectory legs. Periodicity exists, because the order of the planets encountered, the planets' positions, the types of trajectory legs, the hyperbolic excess speeds at the planets, and the minimum passing distances during the encounters repeat or almost repeat periodically.

Interplanetary periodic orbits can be used to make available a scheduled transportation system between two planets. The continuing propulsive requirements of such a system consist of only that needed for guidance and resupply of the vehicles on the periodic orbit.

Hollister[1,2] and Menning[2] found periodic orbits that connect Earth and Venus; however, for periodic orbits connecting Earth and Mars, the small mass of Mars means that much less change in velocity occurs from a close flyby of that planet. This velocity change is necessary to insure that the periodic orbit vehicle can attain the hyperbolic excess velocity vector necessary for the next trajectory leg without propulsive effort and without colliding with the encountered planet. Because the velocity change available at Mars is small, finding a periodic orbit which includes encounters of that planet is more difficult and requires an approach different from that of Hollister and Menning.

Both the method of Hollister and Menning and the method presented below do not guarantee that all periodic orbits connecting the planets of interest will be found. They do, however, supply means of narrowing the search for such orbits.

Method

Periodic swing-by orbits consist of a series of interplanetary trajectory legs that are separated by planetary encounters. There are two types of trajectory legs: interplanetary legs between different planets; and direct returns, which return to the planet from which the trajectory leg last departed. The means by which legs of the two types are combined into a continuous series constitutes a method of searching for periodic orbits. A patched conic analysis, which neglects the finite size of planetary spheres of influence, is used for the search. The resulting orbit must, of course, not collide with an encountered planet.

There are several types of direct returns involving one-half, one, or more revolutions of the vehicle and the planet around the sun. Half-revolution return and full-revolution return trajectories re-encounter the departure planet after completion of one-half a revolution around the sun and a full revolution about the sun, respectively. In addition, there are symmetric return trajectories which return to the departure planet after more than one planetary period and which lie in the plane of the planet's orbit. The symmetry exists in the sense used by Ross[3]: the departure and arrival encounters are symmetrically arranged in space about the line of apsides of the sun-centered ellipse that the symmetric return follows. The departure and arrival speeds are functions of the length of time for each symmetric return. Additional types of direct returns exist but were not included in the investigation.

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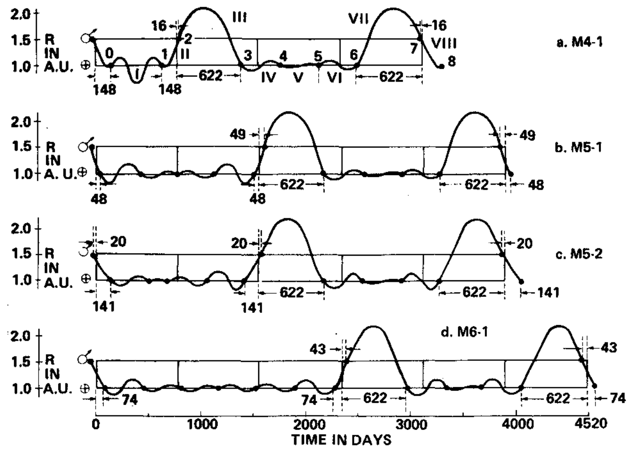

Fig. 1 Distance from the sun as a function of time for several periodic swing-by orbits to Mars.

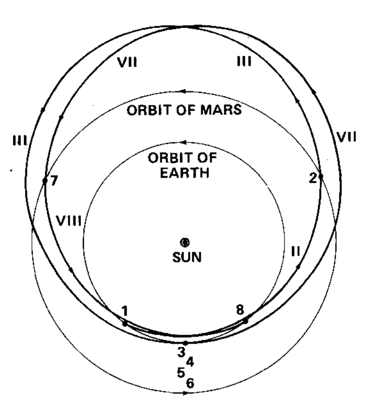

Fig. 2 Interplanetary trajectory legs of orbit $M4$-1 in a sun-centered, inertial coordinate frame.

Hollister's method of finding periodic orbits involves combining two series of direct returns with two interplanetary trajectory legs, one in each direction. Direct returns occur at each of the two planets. Hollister's method does not work for periodic orbits to Mars because of Mars' small mass. The small mass results in insufficient turning of the velocity vector when an attempt is made to match direct returns there. Therefore, a different approach is necessary.

The method used herein requires the existence of a family of Earth-Mars-Earth round trip trajectories and the reciprocal family of round trips. A trajectory that is reciprocal to another one, briefly, is that trajectory whose encounter dates are the negatives of the encounter dates for the original trajectory when all dates are measured relative to the date of opposition. Ross documented such pairs of round trip families and formed trajectory charts for them. A periodic orbit scheme which uses this approach must then include both a round trip trajectory to Mars and its reciprocal. These must be centered around different oppositions of Earth and Mars so that they do not overlap in time. The nature and placement of these reciprocal pairs of round trips is evident in Fig. 1, which plots distance from the sun vs time for several Earth-Mars periodic orbits. Vertical solid lines indicate oppositions of Mars, while large dots indicate planetary encounters. In Fig. 1a, successive encounters are labeled with Arabic numbers, and successive trajectory legs, with Roman numerals. In this figure, one round trip consists of legs II and III, while the reciprocal round trip consists of legs VII and VIII.

Two separate series of direct returns are then required to connect the respective ends of the two round trips to Mars. A list of series of direct returns at Earth was formed (presented in Ref. 4) to help with the selection process. In Fig. 1a, one series of direct returns consists of trajectory leg number I, while the other series consists of legs IV, V, and VI. Both of these series of direct returns center about Earth's orbit instead of one being centered about each planet's orbit.

The ends of the round trips and the direct returns will not match exactly (near points 0, 1, 3 and 6), and a neighboring solution is necessary for the round trips plus any symmetric returns. If the resulting solution indicates that adequate passage distances exist at all planetary encounters, then one has an Earth-Mars periodic orbit for the case of circular, coplanar planetary orbits. Because of the cut-and-try nature of the method, one cannot be certain that all periodic orbits have been found—even among the types of periodic orbits considered. The next step is to look for a solution in the case of elliptic, inclined planetary orbits.

In order to obtain a better approximation to a periodic orbit with an eccentric, inclined model for the solar system, a computer solution is necessary. The computer program first iterates on the encounter dates to equalize the arrival and departure hyperbolic excess speeds at each planetary encounter. After convergence has been achieved, the program calculates the hyperbolic excess velocity turn angles and the passing distances at each encounter. The numerical techniques used are basically those of Menning, although several extensions of his work have been performed to handle half revolution returns and trajectories that travel more than one or two full revolutions around the sun between planetary encounters. Computer solution with the more accurate model for the solar system eliminated several periodic orbit schemes because of inadequate planetary passage distance or because of repeated nonconvergence of the encounter dates.

In no case will a periodic swing-by orbit be found which is truly periodic, because the absolute positions of the planets never repeat exactly, hence, the "periodic orbit" can never repeat exactly. However, in the case of an orbit connecting two planets, a continuous trajectory involving a series of planetary flybys in a basically repeating pattern that is indefinite in length exists at least for a few centuries. Successively better solar system models give successively better approximations to the actual continuous series of flybys. Several periodic orbits have been computed for a solar system model that included accurate values for the planets' eccentricity, relative inclination, period, and location of the ascending node.

The More Promising Periodic Orbits

Only the more promising Earth-Mars periodic orbits are discussed in detail. Promising periodic orbits are those for which the probability is high for the existence of the indefinitely long series of flybys and that provide a large number of round trips to Mars in a given length of time. Less promising periodic orbits have additional, unnecessary direct returns at Earth before visiting Mars again.

The labeling system used here for periodic orbits follows the format Mn-m. n is a number such as 4, 5, or 6 that stands for the number of synodic periods of Earth and Mars before the orbit repeats in the circular, coplanar case. Each orbit found makes two round trips to Mars during this period. m is an arbitrary integer used to differentiate among orbits whose label begins Mn-.

There are, in general, n versions of a periodic swing-by orbit labeled Mn-m, which are distinguished by which Earth-Mars opposition begins the pattern of the orbit. If an $n + 1^{st}$ version is begun at the $n + 1^{st}$ opposition, it will coincide with the extension of the 1^{st} version.

All of the circular, coplanar periodic orbits found in the investigation are presented in Ref. 4. The most promising are periodic orbits $M4$-1, $M5$-1, and $M5$-2. Those periodic orbits numbered $M6$-... or greater can be considered unnecessarily long for the number of round trips to Mars that are achieved.

Table 1 Data on periodic swing-by orbits to Mars

	Periodic orbit								
	*M*4-1			*M*5-1			*M*5-2		
	A	*B*	*C*	*A*	*B*	*C*	A_1/A_2	*B*	*C*
	Hyperbolic excess speed in EMOS								
a)	0.257	0.314	0.181	0.249	0.316	0.211	0.245	0.314	0.183
b)	0.260	0.324	0.195	0.276	0.322	0.216	0.244	0.325	0.195
c)	0.270	0.405	0.229	0.371	0.390	0.260	0.283	0.399	0.230
d)	0.250	0.245	0.178	0.188	0.252	0.171	0.212	0.246	0.172
	Passing distance from planetary center in planetary radii								
a)	1.54	3.77	1.30	1.78	7.30	1.55	1.42/2.06	4.79	1.37
b)	1.40	3.04	3.60	1.65	2.59	1.53	1.32/2.06	5.57	2.64
c)	1.63	7.63	38.7	2.52	6.86	1.76	1.73/2.41	14.2	23.5
d)	1.21	1.12	1.16	1.07	1.00	1.31	1.02/1.81	1.31	1.11
	Turn angle in degrees								
a)	48.3	4.3	77.4	46.0	2.3	60.7	54.1/42.7	3.4	74.8
b)	51.0	6.1	61.1	45.7	7.8	60.8	57.9/43.6	4.4	62.3
c)	57.8	11.2	83.0	80.7	11.1	80.8	67.5/54.8	13.6	86.2
d)	44.6	2.9	6.3	37.2	3.3	51.2	41.4/33.4	1.4	9.9
	Change in encounter date from the circular, coplanar case to the noncoplanar case in days								
e)	−0.8	−0.3	−0.8						
f)	17.7	28.2	5.9						
g)	15.2	25.3	4.7						

A = encounters at Earth next to the short transfers to Mars (*M*4-1: encounters 0 and 1).
B = encounters at Mars (encounters 2 and 7 for *M*4-1).
C = encounters at Earth next to the long transfers to Mars (*M*4-1: encounters 3 and 6).
A_1 = (1 full revolution return next to the half revolution return); A_2 = (2 full revolution returns next to the half revolution return).
a) = circular coplanar.
b) = average; c) = highest; d) = lowest — eccentric, inclined.
e) = average.
f) = root mean square.
g) = average absolute value.

Figure 1 gives distance from the sun as a function of time for periodic orbits *M*4-1, *M*5-1, *M*5-2, and *M*6-1. Figure 2 gives the path of the interplanetary trajectory legs of *M*4-1 in a sun-centered, inertial coordinate frame. Both figures represent the circular, coplanar case. The direct returns are not shown in Fig. 2, because their close coincidence to each other and the orbit of Earth would confuse the picture. Trajectory leg number V in particular is not shown, because it is confined to positions between about 0.18 a.u. above and below the Earth in its orbit. The four numbers at Earth in the "middle" of the trajectory and in the middle of Fig. 2 indicate the four encounters associated with the three full-revolution returns (IV, V, and VI) in the vicinity of Earth's orbit.

Several features are observable from the figures regarding periodic orbits to Mars. First, one should note not only the symmetric arrangement of the round trip segments in each of the periodic orbits but also the symmetric arrangement of the direct returns of orbits *M*4-1, *M*5-1, and *M*6-1 about points in time centered in each series of direct returns. Next, the round trip segments past Mars travel well outside of the orbit of Mars and reach a point beyond 2 a.u. from the sun. Thirdly, the transfers that occur between Earth and Mars near the times of opposition have short times of flight and take up a small percentage of the repeating cycle.

When the eccentric, inclined case is considered, the dates of actual opposition of Earth and Mars change from those of the circular, coplanar case; and the dates of encounter on the periodic orbit change by several days. In addition, the planes of the interplanetary trajectories are inclined to the ecliptic; and hence, the hyperbolic excess velocities at the encountered planets generally increase. The distances of closest approach during flybys are often less, making collision with an encountered planet more likely. Therefore, a periodic orbit that works in the circular, coplanar case will not in all instances work in the eccentric, inclined case.

In addition, the computational problem is much more difficult in the eccentric, inclined case. The main reason for the increased difficulty is the great increase in the number of independent variables (the number of independent encounter dates before the repetition of the periodic orbit). The number of encounter dates is strongly related to the time required for the periodic orbit to approximately repeat. Here, the period of the periodic orbit is much longer than the number of synodic periods indicated by the first integer in its label. The orbit will repeat when both the sequence of encounters of the periodic orbit and the positions of the planets repeat simultaneously. Hence, the period for a periodic orbit (in the noncoplanar case) is obtained by considering two integers: namely, the number of Earth-Mars synodic periods in the basic circular, coplanar encounter sequence (4, 5, or 6) and the number of synodic periods required for the absolute planetary positions to approximately repeat (15 synodic periods in 32 yr). The length of the periodic orbit period in synodic periods is the product divided by the greatest common divisor of these two integers. The *M*5-... orbits are the easiest to calculate (fewest encounter dates), because they have a period of 15 synodic periods or 32 yr. Periodic orbit *M*4-1 is the most difficult, because it has a repeating cycle of 60 synodic periods or 128 yr; the numerical dimension of the problem in terms of the number of independent dates is seventy-five.

The changes in the eccentric, inclined case from the circular, coplanar case are presented in Table 1 for the periodic orbits *M*4-1, *M*5-1, and *M*5-2. Each entry is based on all the encounters in one region of the circular, coplanar period over the complete eccentric, inclined period. The data are based on all of the five possible versions of both *M*5-1 and *M*5-2. Passing distances and turn angles for orbit *M*5-2 for one set of encounters at Earth are given in pairs to reflect the difference in having one or two full revolution returns before or after the half revolution return. The minimum passing distance in this case could be increased in several instances by reordering the direct return trajectory legs. This reordering, of course, requires Earth encounter hyperbolic excess velocity vectors to be retargeted. An important point revealed by the table is that in going to the eccentric, inclined case the characteristics usually change by only several per cent in each case and that the changes do not always increase speeds and reduce minimum passing distances. Lists of encounter dates and speeds for complete, noncoplanar periods are presented in Ref. 4.

An Interplanetary Transportation System Based on Periodic Orbits to Mars

A number of vehicles equal to the number of versions (n = 4, 5, or 6) of a periodic orbit would be necessary to fully utilize all of the available transfers. The n vehicles plus rendezvous shuttle vehicles from the periodic orbit to the planet would provide a regularly scheduled transportation system.

One vehicle for each version of a periodic orbit (4 or more vehicles) provides short transfer times in each direction at each opposition period. Any periodic orbit to Mars includes one short transfer from Earth to Mars and another back to Earth. The sequence of planetary encounters of the orbit requires n synodic periods for completion. Since there are n such sequences covering different oppositions, each time of opposition can be covered by one short transfer between Earth and Mars in each direction.

The n vehicles following the n versions of a periodic orbit could be large and comfortable, and their propulsion requirements could remain small. Once a vehicle has been launched onto a periodic orbit, the continuing propulsion requirement consists only of that needed for guidance. This requirement has been shown to be quite reasonable even with present-day guidance accuracies.[5] Therefore, the fuel requirement will be small even for large vehicles.

Smaller vehicles to shuttle between encountered planets and vehicles on periodic orbits would be necessary to complete the transportation system. These rendezvous shuttles would transfer both personnel and materiel between planetary orbits or surfaces and the vehicles following the versions of the periodic orbit. The ΔV requirements for the smaller shuttle vehicles could be quite high, but these vehicles could be kept small if their primary purpose is to transfer personnel since there would be little need to remain in them for long periods of time.

Not only are the long transfers between Earth and Mars available as transportation, but they could be suited for purposes of interplanetary research between the orbit of Earth and the inner reaches of the asteroid belt. The direct returns at Earth could also be used for research and for purposes of repair and maintenance of the vehicles.

The transportation system discussed here may cost less than that of a one-way transportation system if the purpose of this system is to provide frequent personnel transfer opportunities between Earth and a Martian colony. A disadvantage of this transportation system is that one would be restricted to certain rigid launch times by the periodic orbit.

Conclusions

Periodic orbits to Mars have been shown to exist under the assumptions inherent in patched conic trajectory calculations. Their speeds are similar to those for one-way transfers. They can be used to form a regularly scheduled transportation system between Earth and Mars.

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